Exercise 51

If $f(x) = x^2 + 10 \sin x$, show that there is a number c such that $f(c) = 1000$.

Solution

According to Theorem 7, x^2 and $\sin x$ are continuous at all numbers in their respective domains (all real numbers). By Theorem 4, $10 \sin x$ and $x^2 + 10 \sin x$ are also continuous at all numbers in their respective domains (all real numbers). Find a value of x for which $f(x)$ is less than 1000, and find a value of x for which $f(x)$ is greater than 1000.

$$f(30) \approx 890.12$$

$$f(35) \approx 1220.72$$

$f(x)$ is continuous on the closed interval $[30, 35]$, and $N = 1000$ lies between $f(30)$ and $f(35)$. By the Intermediate Value Theorem, then, there exists a number c such that $f(c) = 1000$.